

EFFECTIVE ELASTIC PROPERTIES OF ALUMINA-ZIRCONIA COMPOSITE CERAMICS PART 5. TENSILE MODULUS OF ALUMINA–ZIRCONIA COMPOSITE CERAMICS

WILLI PABST, GABRIELA TICHÁ, EVA GREGOROVÁ, EVA TÝNOVÁ

*Department of Glass and Ceramics, Institute of Chemical Technology in Prague,
Technická 5, 166 28 Prague 6, Czech Republic*

E-mail: pabstw@vscht.cz

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In this fifth and last paper of a series on the effective elastic properties of alumina-zirconia composite ceramics (AZ composites) the tensile moduli of dense and porous AZ composites are investigated from the theoretical point of view and compared with experimental data. For dense AZ composites the Hashin-Shtrikman bounds turn out to be sufficiently close to each other and excellent agreement is found between theoretically predicted and measured values, so that the arithmetic Hashin-Shtrikman average can be used for predicting effective elastic moduli for arbitrary compositions. For dense zirconia-toughened alumina (ZTA) with 15 wt.% and dense alumina-containing tetragonal zirconia (ATZ) with 80 wt.% of zirconia the theoretically predicted effective tensile moduli are 375 GPa and 251 GPa, respectively. For porous AZ composites (prepared by starch consolidation casting) the consistency of the experimentally measured data is assessed with regard to the Hashin-Shtrikman upper bound. Fitting results confirm the superiority of the new relation $E/E_0 = (1 - \phi)(1 - \phi/\phi_c)$, where E is the effective tensile modulus and ϕ the porosity, over most other fit models. Extrapolated E_0 values are 351 GPa and 237 GPa and critical porosities ϕ_c are 0.796 and 0.882 for porous ZTA and ATZ, respectively.

INTRODUCTION

In Parts 1 and 2 of this series of papers the linear theory of elasticity has been summarized for anisotropic and isotropic materials [1], the fundamentals of micromechanical modeling have been reviewed and rigorous bounds, dilute approximations and nonlinear relations have been given [2]. In Parts 3 and 4 the effective elastic moduli of polycrystalline alumina and zirconia have been calculated from monocrystal data and compared with published values [3] and measured data for porous alumina and zirconia have been analyzed using several fit models, including newly proposed ones [4]. It is the purpose of the present paper to bring all this knowledge together and to predict, analyze and discuss the effective tensile moduli of alumina-zirconia composite ceramics (AZ composites), dense as well as porous. In the experimental section the materials and their processing (slip casting and starch consolidation casting for dense and porous AZ composites, respectively) are briefly described and the measurement methods applied are mentioned (tensile modulus via the resonant frequency method, porosity determination via the bulk density measured by the Archimedes method). In the first part of the results section the effective theoretic

density and the effective elastic moduli of dense AZ composites of arbitrary composition are discussed in some detail and handy formulae are given for general use. In the second part of the results section the effective tensile modulus of two typical AZ composites is investigated in dependence of the porosity: zirconia-toughened alumina with a zirconia content of 15 wt.% (labelled here ZTA) and alumina-containing tetragonal zirconia with a zirconia content of 80 wt.% (labelled here ATZ). The effective tensile modulus is measured by the resonant frequency method and its porosity dependence is analyzed using 11 different fit models.

EXPERIMENTAL

Dense alumina and alumina-zirconia composite ceramics (AZ composites) were prepared by co-milling mixed aqueous suspensions of submicron alumina and zirconia powders in a high-energy planetary mono mill (Fritsch Pulverisette 6, Germany) and subsequent slip casting of these suspensions (containing 75 wt.% solids) into plaster molds. After demolding and drying the AZ composites were fired at 1530°C. Further processing details can be found elsewhere [5-7].

The powder types used are AA04 (Sumitomo, Japan), a very pure (>99.99 wt.% Al_2O_3) powder with highly isometric grain shape and a median particle size of $D_{50} = 0.4 \mu\text{m}$ and TZ-3Y (Tosoh, Japan), a tetragonal zirconia type with a median particle size of $D_{50} = 0.3 \mu\text{m}$, containing 3 mol.% (5 wt.%) Y_2O_3 . Microstructural investigation by image analysis on thermally etched polished sections revealed a maximum alumina grain size of $1.3 \mu\text{m}$ ($\pm 0.5 \mu\text{m}$) and a maximum zirconia grain size of $0.6 \mu\text{m}$ ($\pm 0.3 \mu\text{m}$) in the AZ composites after firing. Note that the alumina grain size in AZ composites is significantly smaller than in (identically processed) pure alumina, which can be $2 \mu\text{m}$ ($\pm 1 \mu\text{m}$), an effect which is attributed to the mechanical constraint exerted by the zirconia grains on the alumina grains, which inhibits alumina grain growth, cf. [5-7]. Depending on the mixture composition we denote the AZ composites by the simple and self-evident notation A90Z10, A80Z20 etc., which denote AZ composites with 90 wt.% alumina and 10 wt.% zirconia, AZ composites with 80 wt.% alumina and 20 wt.% zirconia and so on.

Porous AZ composites were prepared from commercial mixed powders by starch consolidation casting (SCC) from aqueous suspensions containing 70-76 wt.% solids and nominal starch contents of 5, 10, 15, 30 and 50 vol.% (with respect to the oxide powder content, based on an approximate native starch density of 1.50 g/cm^3). The suspensions were cast into brass molds and heated up to approx. 80°C in order to initiate swelling of the starch grains and subsequent gelation of the suspension. After demolding and drying the AZ composites were fired at 1530°C and 1550°C , the lower temperature referring the composites with higher zirconia content. Further processing details can be found elsewhere [8], cf. also [9-14].

Two special, though widely used and thus typical, compositions of AZ composites were selected for the SCC process, A85Z15 (labelled ZTA in the following) and A20Z80 (labelled ATZ in the following). Commercial powder mixtures were used: ZTA-85 (Daiichi, Japan), a mixed powder with 15 wt.% zirconia (tetragonal zirconia containing 3 mol.% Y_2O_3) and ATZ-80 (Daiichi, Japan), a mixed powder with 80 wt.% zirconia, both submicron with a median particle size of $D_{50} = 0.3 \mu\text{m}$. The starch used is native potato starch (Naturamyl, Czech Republic) with a median size of approx. $D_{50} = 50 \mu\text{m}$ (starch globules in the native state, i.e. before swelling) and oval, almost spherical shape. The pore size resulting from the starch remains approx. $50 \mu\text{m}$ after firing. Interestingly, (linear) firing shrinkage is independent of the starch content and depends only on the alumina-zirconia ratio, i.e. the oxide composition (15-18 % for ZTA and 23-26 % for ATZ), cf. [8]. This is a clear indication of the fact that the starch-pores do not

shrink during the sintering process. Thus, it can be expected that after firing the pores are almost 2 orders of magnitude larger than the grain size.

The as-fired specimens were of cylindrical shape with a diameter of approx. 4 mm and a length between 50 and 80 mm. The bulk density ρ was determined by the Archimedes method and the total porosity ϕ was calculated from the bulk density and the theoretical density ρ_0 according to the standard formula

$$\phi = 1 - \frac{\rho}{\rho_0} \quad (1)$$

where the theoretical density of alumina and (tetragonal) zirconia was assumed to be 4.0 g/cm^3 and 6.1 g/cm^3 , respectively (for AZ composites see below). The effective tensile modulus of the porous ceramics was measured at the Institute of Rock Structure and Mechanics (Academy of Sciences of the Czech Republic) by the resonant frequency technique, using the resonant frequency tester Erudite (CNS Electronics, UK) in the frequency range 0-100 kHz and calculated via the formula for resonant frequencies of longitudinal vibrations

$$c = \sqrt{\frac{E}{\rho}} \Rightarrow E = \rho \cdot (2LF)^2 \quad (2)$$

where $c = 2LF$ is the sound velocity, E is the effective tensile modulus, ρ the bulk density, L the specimen length and F the resonant frequency [15,16]. To apply this formula it is recommended that the length-to-diameter ratio of the specimens should be between 5 and 20 [17,18], which is fulfilled in our case.

Fitting of the $E - \phi$ - dependence data sets was performed using the non-linear regression software package CurveExpert 1.3 (Danial Hyams, USA), without initial guesses for the fit parameters wherever possible (by default, all fit parameters were set equal to unity).

RESULTS AND DISCUSSION

Dense Alumina-Zirconia Composite Ceramics

Figure 1 shows the effective theoretical density of alumina-zirconia composite ceramics (AZ composites) as a function of the zirconia weight fraction X , calculated¹ under the assumption that all zirconia is tetragonal (solid line) and monoclinic (dashed line), respectively. The constituent phase densities have been assumed to be 4.0 g/cm^3 (alumina), 6.1 g/cm^3 (t-zirconia) and 5.6 g/cm^3 (m-zirconia). To the precision required, these are almost universally accepted values.² The extreme case of 100 % m-zirconia should be considered as illustrative. It should serve in order to assess the possible error introduced into the calculations when part of the t-zirconia has transformed to m-zirconia, e.g. due to

hydrothermal ageing (surface degradation at moderately elevated temperature in aqueous environments) [19-23]. For an as-prepared AZ composite, which has not yet undergone degradation, the t-zirconia content is usually close to 100 % (with respect to the total zirconia content).

Third-order polynomials can be used to fit the $\rho_0 - X$ dependences (with correlation coefficients >0.99999). In the extreme case of AZ composites with 100 % t-zirconia we obtain

$$\rho_0(X) = 4.0 + 1.41 X + 0.3 X^2 + 0.4 X^3 \quad (3)$$

while in the case of AZ composites with 100 % m-zirconia we have

$$\rho_0(X) = 4.0 + 1.10 X + 0.3 X^2 + 0.2 X^3 \quad (4)$$

These handy formulae (with all terms in g/cm^3) can be used to calculate the theoretical density of a (classical, i.e. non-nano) AZ composite of arbitrary composition when the zirconia weight fraction is given.

Trivially, the corresponding dependences of the theoretical density ρ_0 on the zirconia volume fraction ϕ_z are given by simple linear mixture rules (cf. the straight lines in figure 2),

$$\rho_0(\phi_z) = 4.0 + 2.1 \phi_z \quad (5)$$

$$\rho_0(\phi_z) = 4.0 + 1.6 \phi_z \quad (6)$$

for the extreme cases of AZ composites with 100 % t-zirconia and 100 % m-zirconia, respectively. With the help of these formulae (or, alternatively, figures 1 and 2) it can easily be verified that under the assumption of 100 % prevalence of t-zirconia the zirconia volume fraction is 0.105 for the ZTA composite (corresponding to $X = 0.15$) and 0.726 for the ATZ composite (corresponding to $X = 0.80$).

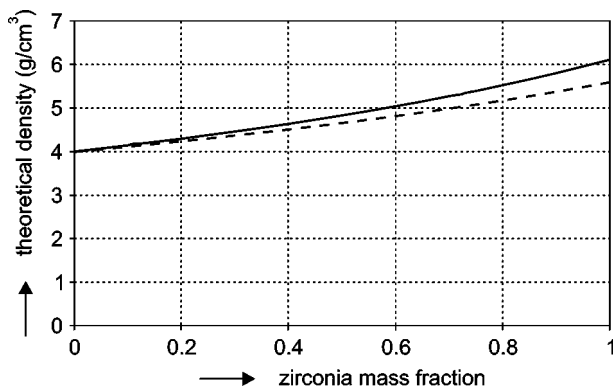


Figure 1. Effective theoretical density of AZ composites as a function of the zirconia weight fraction, calculated for a two-phase composite under the assumption that all zirconia is tetragonal (solid line) and that all zirconia is monoclinic (dashed line), respectively; constituent phase densities 4.0 g/cm^3 (alumina), 6.1 g/cm^3 (t-zirconia) and 5.6 g/cm^3 (m-zirconia).

In order to estimate or predict the effective elastic moduli of dense AZ composites we use the E , G and K values recommended in [3] for alumina and t-zirconia containing 3 mol.% Y_2O_3 (the "pure" end members of AZ composites), respectively. These are $E = 400 \text{ GPa}$, $G = 163 \text{ GPa}$, $K = 247 \text{ GPa}$ (alumina) and $E = 210 \text{ GPa}$, $G = 80 \text{ GPa}$, $K = 184 \text{ GPa}$ (zirconia). Using the E values, the Voigt-Reuss bounds [2] for the effective tensile moduli of dense AZ composites can be readily calculated. These are shown as solid lines in figure 3, in which experimentally determined effective tensile moduli of dense AZ composites (with porosity $< 3\%$) prepared by slip-casting are also shown. It is evident that all measured values (cf. table 1) fall within the Voigt-Reuss bounds. Although the Voigt-Reuss bounds are rather close in the case of dense AZ composites, an even more precise estimate (prediction) can be given via the Hashin-Shtrikman bounds. Under the assumption of statistical isotropy (cf. [2]), which is justified here, the Hashin-Shtrikman bounds for the elastic moduli of dense AZ composites can be calculated via the relations given in [2]. The results are shown as dashed lines in figure 3 and are on the one hand very close to each other and on the other hand in very good agreement with the measured data. This indicates that the precision of the measuring method is excellent and that the scatter observed for porous ceramics (cf. also [4] and below) reflects random variations in microstructure.

Evidently, the arithmetic mean³ of the Hashin-Shtrikman bounds is an excellent estimate for the effective elastic moduli of AZ composite of arbitrary composition when the zirconia volume fraction ϕ_z is given. The resulting dependences can be fitted with second-order polynomials (with correlation coefficients >0.99995) to obtain the following handy formulae (with all terms in GPa).

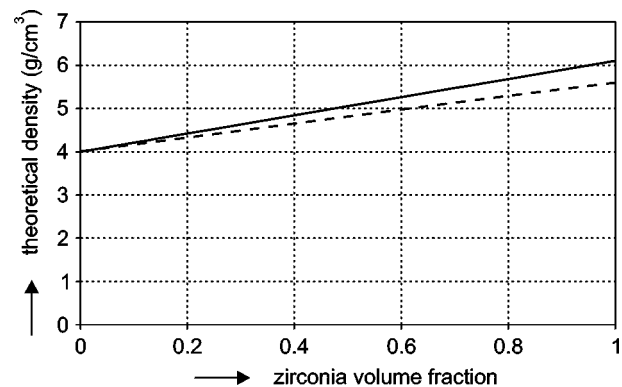


Figure 2. Effective theoretical density of AZ composites as a function of the zirconia volume fraction for the case that all zirconia is tetragonal (solid line) and that all zirconia is monoclinic (dashed line), respectively; constituent phase densities 4.0 g/cm^3 (alumina), 6.1 g/cm^3 (t-zirconia) and 5.6 g/cm^3 (m-zirconia).

Table 1. Measured (without asterisk) and calculated (with asterisk *) effective tensile moduli of dense (porosity < 3 %) alumina, zirconia and AZ composites (prepared by slip-casting); the estimated measurement error is between ±2 and ±4 GPa.

	Zirconia weight fraction X	Zirconia volume fraction ϕ_z	Tensile modulus E (GPa)
Pure alumina	0	0	399 (400 *)
A90Z10	0.10	0.069	385
A85Z15 (ZTA)	0.15	0.105	376 (375*)
A80Z20	0.20	0.142	365
A75Z25	0.25	0.180	353
A70Z30	0.30	0.219	350
A60Z40	0.40	0.304	333
A20Z80 (ATZ)	0.80	0.726	255 (251*)
Pure zirconia	1	1	210

Effective tensile modulus of dense AZ composites:

$$E = 400 - 245.9 \phi_z + 56.3 \phi_z^2 \quad (7)$$

Effective shear modulus of dense AZ composites:

$$G = 163 - 110.4 \phi_z + 28.3 \phi_z^2 \quad (8)$$

Effective bulk modulus of dense AZ composites:

$$K = 247 - 73.6 \phi_z + 10.8 \phi_z^2 \quad (9)$$

According to these formulae, the best estimates (predictions) for effective elastic moduli of AZ composites are $E = 375$ GPa, $G = 151$ GPa, $K = 239$ GPa for the ZTA composite (corresponding to $X = 0.15$ and thus $\phi_z = 0.105$) and $E = 251$ GPa, $G = 97$ GPa, $K = 199$ GPa for the ATZ composite (corresponding to $X = 0.80$ and thus $\phi_z = 0.726$).

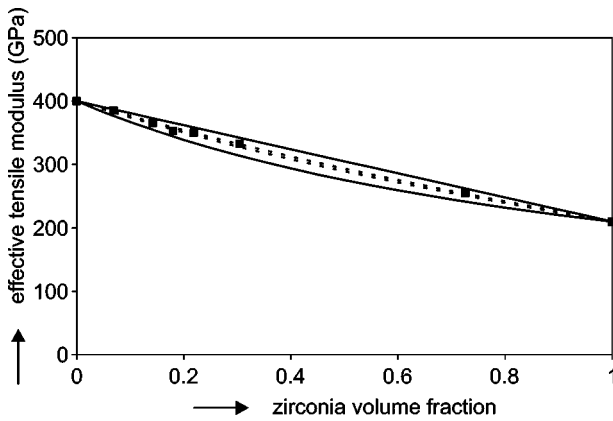


Figure 3. Effective tensile moduli of dense (porosity < 3 %) AZ composites as a function of the zirconia volume fraction; Voigt-Reuss bounds (solid lines), Hashin-Shtrikman bounds (dashed lines) and data measured by the resonant frequency technique for AZ composites prepared by slip-casting (full squares).

Porous Alumina-Zirconia Composite Ceramics

The effective tensile moduli of two types of AZ composites (ZTA and ATZ), both prepared by starch consolidation casting (SCC) are listed in tables 2 and 3 for five different porosities. Figures 4 and 5 show the measured data, together with the Voigt bound and the Hashin-Shtrikman upper bound.⁴ Although none of the two data sets violates the bounds, neither the Voigt bound nor the Hashin-Shtrikman upper bound, the ZTA data exhibit a steep increase with decreasing porosity, possibly due to an error in the porosity determination (for the data point with $\phi = 0.166$).

It has to be noted that the application of the rigorous upper bounds for two-phase materials (porous media) in this case involves the tacit assumption that the solid AZ composite matrix (skeleton) can be replaced

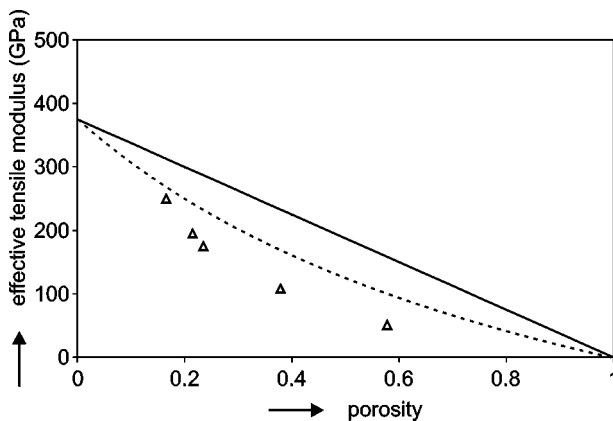


Figure 4. Porosity dependence of the effective tensile moduli of ZTA composites; Voigt bound (solid line), Hashin-Shtrikman upper bounds (dashed line) and data measured by the resonant frequency technique for ZTA composites prepared by starch consolidation casting with potato starch (empty triangles).

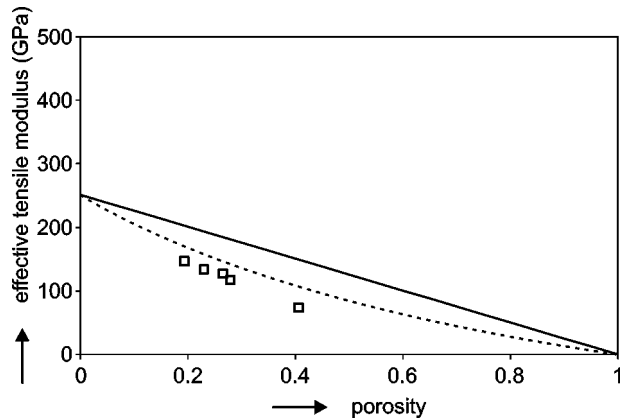


Figure 5. Porosity dependence of the effective tensile moduli of ATZ composites; Voigt bound (solid line), Hashin-Shtrikman upper bounds (dashed line) and data measured by the resonant frequency technique for ATZ composites prepared by starch consolidation casting with potato starch (empty squares).

by a "quasi-one-phase" continuum (effective medium at the microscale) with effective properties "smeared out" with respect to the length scale of the pores (macro-scale). This assumption is justified only because the pore sizes are 1-2 orders of magnitude larger (tens of microns) than the alumina and zirconia grain sizes (approx. 1 μm). Otherwise the material would have to be modeled as a three-phase composite, for which the Hashin-Shtrikman bounds (in case one phase is the pore phase, only the Hashin-Shtrikman upper bound) cannot be written in the simple explicit form given in [2]. Since our model assumes the existence of two quite distinct length scales it could be termed hierarchical "pseudo-binary" composite model.

Table 2. Measured effective tensile moduli of porous ZTA composite ceramics prepared by SCC.

Porosity ϕ	Tensile modulus E (GPa)
0.166	250
0.215	195
0.235	175
0.379	108
0.578	51

Table 3. Measured effective tensile moduli of porous ATZ composite ceramics prepared by SCC.

Porosity ϕ	Tensile modulus E (GPa)
0.193	147
0.230	134
0.265	127
0.279	117
0.406	74

In analogy to Part 4 of this series of papers [4], the $E - \phi$ - dependence for ZTA and ATZ can be fitted using the 11 relations (fit models) introduced in [2]. For theoretical background and a more detailed discussion of the individual relations cf. [24-29]. For easy reference, we repeat the relations here.

$$\text{Model 1} \quad \frac{E}{E_0} = \exp(-2\phi) \quad (10)$$

$$\text{Model 2} \quad \frac{E}{E_0} = \exp(-B\phi) \quad (11)$$

$$\text{Model 3} \quad \frac{E}{E_0} = \exp\left(\frac{-2\phi}{1-\phi}\right) \quad (12)$$

$$\text{Model 4} \quad \frac{E}{E_0} = \exp\left(\frac{-B\phi}{1-\phi}\right) \quad (13)$$

$$\text{Model 5} \quad \frac{E}{E_0} = \exp\left(\frac{-2\phi}{1-\phi/\phi_c}\right) \quad (14)$$

$$\text{Model 6} \quad \frac{E}{E_0} = \exp\left(\frac{-B\phi}{1-\phi/\phi_c}\right) \quad (15)$$

$$\text{Model 7} \quad \frac{E}{E_0} = (1-\phi)^2 \quad (16)$$

$$\text{Model 8} \quad \frac{E}{E_0} = (1-\phi)^N \quad (17)$$

$$\text{Model 9} \quad \frac{E}{E_0} = (1-\phi/\phi_c)^N \quad (18)$$

$$\text{Model 10} \quad \frac{E}{E_0} = \frac{(1-\phi)}{(1-\phi/\phi_c)} \quad (19)$$

$$\text{Model 11} \quad \frac{E}{E_0} = (1-\phi) \cdot (1-\phi/\phi_c) \quad (20)$$

In all these relations E_0 is the tensile modulus of the dense (i.e. pore-free) ceramic and, except for the Haselmann relation (Model 10), the fit parameter ϕ_c can in principle be assigned the physical meaning of a critical porosity, for which the integrity of the structure breaks down, i.e. $E = 0$ for $\phi = \phi_c$. The fit parameters are listed and commented in tables 4 through 7.

As expected, the simple exponential relations (Models 1 and 2) yield unphysical behavior in the high-porosity limit ($E > 0$ for $\phi = 1$ and violation of the Voigt bound for porosities close to unity). Relatively reasonable behavior is exhibited by the modified exponential relations (Models 3 and 4), while the Mooney-type relations (Models 5 and 6) are either unsuccessful in fitting or need initial guesses and nevertheless lead to artefacts (E_0 and B too high, ϕ_c unphysical, partially convex fit curve with inflection point). Models 7 and 8 are rather reasonable, with Model 7 (one parameter) being more robust and Model 8 (two parameters) yielding better fits.

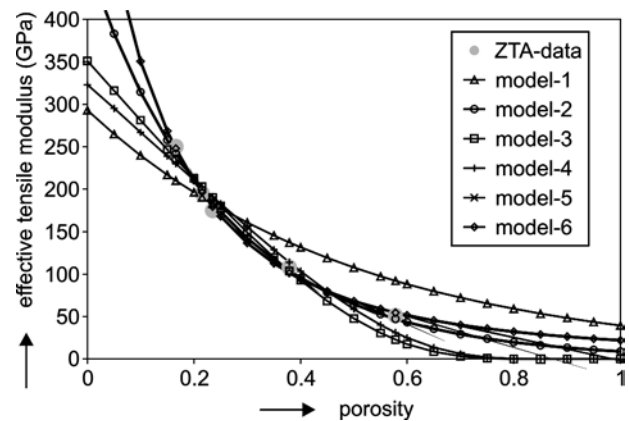


Figure 6. Tensile modulus porosity dependence fitted by exponential models (ZTA composite ceramics prepared by SCC).

Table 4. Fit parameters (E_0 in GPa) for the tensile modulus porosity dependence fitted by exponential models (ZTA composite ceramics prepared by SCC).

	Corr. coeff.	E_0	ϕ_c	B	Initial guess	Remark
Model 1	0.908	292.8	–	–	–	
Model 2	0.996	466.8	–	3.959	–	E_0 too high
Model 3	0.972	351.1	–	–	–	
Model 4	0.976	322.9	–	1.706	–	
Model 5	–	–	–	–	–	fit unsuccessful
Model 6	0.998	645.9	-1.011	6.719	$E_0 = 375$	E_0 and B too high, ϕ_c unphysical

Table 5. Fit parameters (E_0 in GPa) for the tensile modulus porosity dependence fitted by power-law and related models (ZTA composite ceramics prepared by SCC).

	Corr. coeff.	E_0	ϕ_c	N	Initial guess	Remark
Model 7	0.975	322.5	–	–	–	
Model 8	0.989	380.0	–	2.626	–	
Model 9	0.996	466.8	47504	188060	–	E_0 and N too high, ϕ_c unphysical
Model 10	0.999	756.9	-0.11	–	–	E_0 too high, ϕ_c unphysical
Model 11	0.985	351.1	0.796	–	–	

Table 6. Fit parameters (E_0 in GPa) for the tensile modulus porosity dependence fitted by exponential models (ATZ composite ceramics prepared by SCC).

	Corr. coeff.	E_0	ϕ_c	B	Initial guess	Remark
Model 1	0.933	207.4	–	–	–	
Model 2	0.988	273.6	–	3.089	–	
Model 3	0.965	249.3	–	–	–	
Model 4	0.997	213.4	–	1.530	–	inflection point
Model 5	–	–	–	–	–	fit unsuccessful
Model 6	0.997	198.6	0.792	1.186	$E_0 = 251$	inflection point

Table 7. Fit parameters (E_0 in GPa) for the tensile modulus porosity dependence fitted by power-law and related models (ATZ composite ceramics prepared by SCC).

	Corr. coeff.	E_0	ϕ_c	N	Initial guess	Remark
Model 7	0.991	226.2	–	–	–	
Model 8	0.994	238.9	–	2.188	–	
Model 9	0.988	273.6	-24516	-75730	–	ϕ_c and N unphysical
Model 10	0.987	293.1	-0.34	–	–	ϕ_c unphysical
Model 11	0.994	237.0	0.882	–	–	

The three-parameter Phani-Niyogi relation (Model 9), although delivering fits with high correlation coefficients, can be said to have failed in both cases, due to unrealistic values of the fit parameters. The same can be said of the Hasselman relation (Model 10). Of all relations tested the newly proposed Model 11 turned out to be the most useful tool for fitting $E - \phi$ data. This finding confirms the comparative results for porous alumina and zirconia in Part 4 of this series of papers [4] and seems to hold for all porous materials where deviations from the isometric pore shape are negligible.

The intrinsic tensile moduli $[E]$, as determined from Models 4 and 8, are between -1.7 and -2.6 for ZTA and between -1.5 and -2.2 for ATZ. The considerable scatter in these values around the spherical pore value of $[E] = -2$ might be caused by the fact that the data are too rough for deviations from isometric pore shape to be detected. In any case these fitting results yield no indications of anisometric pore shape in our samples.

The values of E_0 and ϕ_c determined by Model 11 are 351 GPa and 0.796 and 237 GPa and 0.882 for porous ZTA and porous ATZ composite ceramics, re-

spectively. The ϕ_c values are reasonable from a physical viewpoint and correspond well to the visual inspection of the graphs. The E_0 values are in good agreement with those determined via the other reasonable models where fitting was successful (Models 3, 4, 7 and 8), viz. $E_0 = 344 \pm 24$ GPa and $E_0 = 232 \pm 13$ GPa for ZTA and ATZ, respectively. There seems, however, to be a general tendency of extrapolated E_0 values of being slightly underestimated (the theoretical values for dense ceramics are 375 GPa for ZTA and 251 GPa for ATZ, see above). The same was found for porous zirconia in [4]. With respect to the high precision of the resonant frequency method this phenomenon might be attributed to a tendency of determining the porosity to high. This can happen when a certain amount of zirconia has transformed into the monoclinic phase and thus the theoretical density in equation (1) is assumed higher than it actually is. Nevertheless, the ultimate reason of this finding must remain a subject of further research.

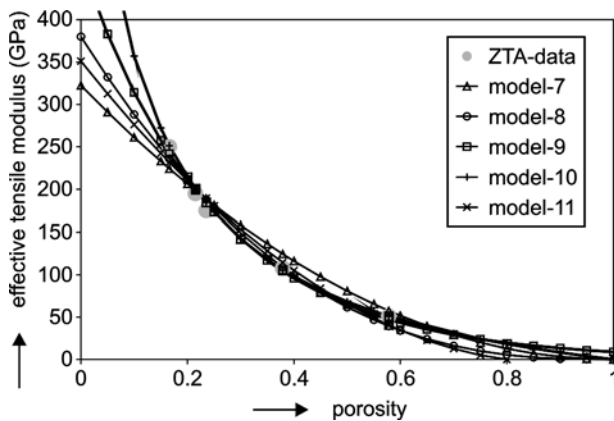


Figure 7. Tensile modulus porosity dependence fitted by power-law and related models (ZTA composite ceramics prepared by SCC).

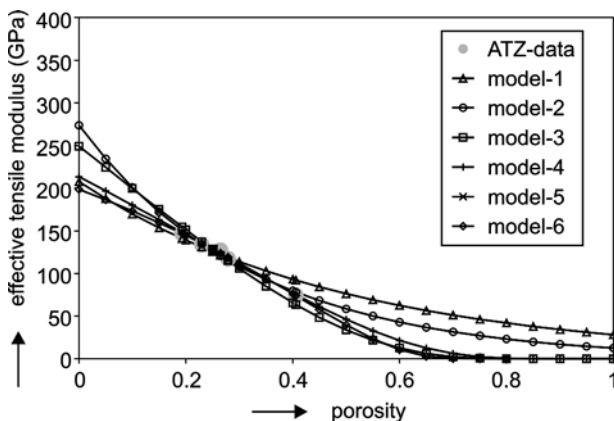


Figure 8. Tensile modulus porosity dependence fitted by exponential models (ATZ composite ceramics prepared by SCC).

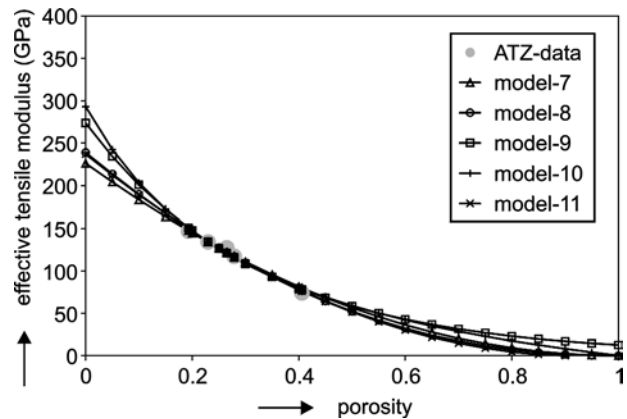


Figure 9. Tensile modulus porosity dependence fitted by power-law and related models (ATZ composite ceramics prepared by SCC).

CONCLUSIONS

In this fifth and last paper of a series on the effective elastic properties of alumina-zirconia composite ceramics (AZ composites) the tensile modulus of dense and porous AZ composites has been investigated, both theoretically and experimentally. For dense AZ composites the Hashin-Shtrikman bounds turned out to be sufficiently close to each other and excellent agreement was found between theoretically predicted and measured values, so that the arithmetic average of the Hashin-Shtrikman bounds can be used for predicting effective elastic moduli for arbitrary compositions. Handy formulae have been given for the fast calculation of effective theoretical densities and effective elastic moduli of dense AZ composites, cf. equations (3) through (9). For dense zirconia-toughened alumina (ZTA) with 15 wt.% and dense alumina-containing tetragonal zirconia (ATZ) with 80 wt.% of zirconia the theoretically predicted effective tensile moduli are 375 GPa and 251 GPa, respectively.

Adopting a hierarchical "pseudo-binary" composite model, i.e. based on the assumption that the solid AZ composite matrix (skeleton) can be replaced by a "quasi-one-phase" continuum (at the microscale) with effective properties "smeared out" with respect to the length scale of the pores (macroscale), the porosity dependence of the effective tensile modulus has been analyzed for ATZ and ZTA (prepared by starch consolidation casting). The consistency of the experimentally measured data has been assessed with regard to the Hashin-Shtrikman upper bound. Fitting results confirm the superiority of the new relation $E/E_0 = (1 - \phi) \cdot (1 - \phi/\phi_c)$, where E is the effective tensile modulus and ϕ the porosity, over most other fit models. Extrapolated E_0 values are 351 GPa and 237 GPa and critical porosities ϕ_c are 0.796 and 0.882 for porous ZTA and ATZ, respectively.

Footnotes:

¹ For a classical two-phase composite, in which the phase boundaries play no role in determining the density and other effective properties. Note that these and the following calculations cannot be assumed to be valid for nanocomposites.

² When higher precision is required the type and content of lattice-stabilizing agents (yttria, ceria, magnesia, calcia etc.) must necessarily be specified.

³ Other means, e.g. the harmonic or geometric mean, would do just as well. Due to the proximity of the Hashin-Shtrikman bounds in our case, the difference between the means is absolutely negligible here.

⁴ Of course, the Reuss bounds and the Hashin-Shtrikman lower bounds are zero in this case, cf. [2].

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MAKROSKOPICKÉ ELASTICKÉ VLASTNOSTI
KOMPOZITNÍ KERAMIKY NA BÁZI Al_2O_3 A ZrO_2
ČÁST 5. YOUNGOVY MODULY KOMPOZITNÍ
KERAMIKY NA BÁZI Al_2O_3 A ZrO_2

WILLI PABST, GABRIELA TICHÁ,
EVA GREGOROVÁ, EVA TÝNOVÁ

Ústav skla a keramiky,
Vysoká škola chemicko-technologická v Praze,
Technická 5, 166 28 Praha 6, Česká republika

V této páté a poslední části z řady prací zabývajících se makroskopickými elastickými vlastnostmi kompozitní keramiky na bázi Al_2O_3 a ZrO_2 („AZ kompozitů“) je studován Youngův modul hutných (tj. neporézních) resp. porézních AZ kompozitů jak z teoretického tak z experimentálního hlediska. V případě hutných AZ kompozitů jsou Hashin-Shtrikmanovy meze dostatečně blízko u sebe a vynikající shoda byla nalezena

mezi teoreticky vypočtenými a experimentálně naměřenými hodnotami. Tzn. že aritmetický průměr Hashin-Shtrikmanových mezí může být používán pro předpověď efektivních elastických modulů u AZ kompozitů libovolného složení. V této práci jsou uvedeny praktické a užitečné vztahy pro rychlý výpočet efektivních teoretických hustot a efektivních elastických modulů hutných AZ kompozitů - rovnice (3) až (9). Pro hutnou ZTA keramiku s 15 hm.% ZrO_2 a hutnou ATZ keramiku s 80 hm.% ZrO_2 jsou teoreticky vypočtené efektivní Youngovy moduly 375 GPa resp. 251 GPa.

Na základě hierarchického modelu „pseudo-binárního“ kompozitu byla dále analyzována závislost efektivního Youngova modulu na pórovitosti pro ZTA a ATZ keramiku připravenou tzv. škrobovým litím. Experimentálně naměřená data jsou porovnána s horní Hashin-Shtrikmanovou mezí. Výsledky fitování potvrzují užitečnost nového vztahu $E/E_0 = (1 - \phi)(1 - \phi/\phi_c)$, kde E je efektivní Youngův modul a ϕ pórovitost. Extrapolované hodnoty E_0 jsou 351 GPa resp. 237 GPa a kritické pórovitosti ϕ_c jsou 0.796 resp. 0.882 pro porézní ZTA resp. ATZ.

